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Dyons in presence of gravitation and symmetrized field equations

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Abstract : Combined theory of gravitation and electromagnetism associated with particles carrying electric and magnetic charges has been established from an invariant action principle. Corresponding field equations, equation of motion and Einstein Maxwell's equations are obtained in unique and consistent way. It is shown that weak field approximation of slowly moving particle in gravitational field leads the symmetry between electromagnetic and linear gravitational fields. Postulation of the existence of gravimagnetic monopole leads structural symmetry between generalized electromagnetic and gravielectromagnetic fields. Corresponding quantization conditions and angular momentum are also analysed.

Keywords : Dyons, Maxwell's-Dirac equations, generalized electromagnetic fields, gravitation

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1. Introduction

Magnetic monopoles were introduced by Dirac [1] to symmetrize Maxwell's equation in a manifest way. Monopoles of lowest mass are expected to be stable since magnetic charge should be conserved like electric charge. Observation made by Cabrera [2] has arosed the interests of many to study monopoles and their potential importance in connection with quark confinement [3], magnetic condensation of vaccum [4], CP-voilation [5], proton-decay [6], current grand unified theories [7] and supersymmetry [8]. Despite the enormous potential importance the formalism necessary to describe them has been clumsy and manifestly non-covariant. The quantum mechanical excitation of fundamental monopoles include dyons [9,10] which are automatically arisen from the semi-classical quantization of global charge rotation degrees of freedom of monopoles. In connection with CP-voilation [5] in terms of nonzero angle of vaccum monopoles are necessarily dyons and Dirac quantization permits them to have an anamolous electric charge. Existence of such dyons removes the objection of spin-statistics relationship [11] for monopoles and also those raised by Pantaleone [12] towards experimental results of Fairbank *et al* [13] about the reported evidence of fractional electric charge.

Introducing the idea of two potentials [14] to avoid controversial string variables [1], Rajput *et al* [15,16] constructed the manifestly covariant field theory of dyons. These dyons carry generalized charge, potential, current, vector field and antisymmetric field tensor as complex quantities with electric and magnetic quantities as real and imaginary constituents. On the other hand in the case of gravitation attempts are made towards the quantization of gravitational charge [17,18]. The postulation of Heavisidian (gravimagnetic) monopoles [19–21] has increased the structural symmetry between the linear equations of gravitational field and the extended Maxwell's equations. Keeping these facts in mind, in the present paper, we have made an attempt to investigate the unified theory of general relativity in presence of electromagnetic fields associated with dyons. Starting from an invariant action principle we have derived the covariant field equations, equations of motions and Einstein field equations. Combined theory of gravitation and generalized electromagnetism has been reformulated in covariant and consistent way. Considering the weak field approximation for slowly moving particles in gravitational field it is emphasized that theories of linear gravity and electromagnetism play symmetric roles. Postulating the existence of gravimagnetic monopoles, Maxwell like equations and equations of motion of linear gravity are derived. Demonstrating the structural symmetry between the generalized electromagnetic fields of dyons and those for gravielectromagnetic fields of gravito-dyons, charge (mass) quantization condition has been discussed and corresponding angular momentum operator is analysed. It has been pointed out that gravimagnetic poles are most strongly interacting form of matter.

2. Invariant action principle and dyon field equations in presence of gravitation

Let us begin with a physical system whose dynamical equations are derived from "Principle of least-action". We define the following action [22] for a system of n particles consisting mass m_n, electric charge e_n magnetic charge g_n, and electromagnetic field tensors $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ (associated with dyons) in presence of gravitation as :

$$I_p = I_e + I_F + I_{\text{in}}, \tag{1}$$

where

$$I_e = -\sum_n m_n \int_{-\infty}^{+\infty} d\tau\left[-g_{\mu\nu}(x)(dx^\mu / d\tau)(dx_n^\nu / dt)\right]^{1/2} \tag{1a}$$

$$I_F = -\tfrac{1}{4}\int d^4x(-g)^{1/2}\left[F_{\mu\nu}(x)F^{\mu\nu}(x)+\tilde{F}_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)\right] \tag{1b}$$

$$I_{\text{int}} = \sum_n e_n\int_{-\infty}^{+\infty} d\tau(dx_n^\mu / d\tau)A_\mu(x)+\sum_n g_n\int_{-\infty}^{+\infty} d\tau(dx_n^\mu / d\tau)B_\mu(x) \tag{1c}$$

are the contributions respectively due to particle, field and interaction term. The subscript p denotes that the action is described for the matter and the radiation while the $g_{\mu\nu}(x)$

represents the usual gravitational field. $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ (given in eq. 1(b)) are generalized electromagnetic field tensors associated with electric and magnetic charges. These field tensors are described as

$$F_{\mu\nu} = E_{\mu\nu} - \tilde{H}_{\mu\nu}, \qquad F_{\mu\nu} = H_{\mu\nu} + \tilde{E}_{\mu\nu};$$
$$E_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}, \qquad H_{\mu,\nu} = B_{\mu,\nu} - B_{\nu,\mu}; \tag{2}$$
$$\tilde{E}_{\mu\nu} = \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma}E^{\rho\sigma}, \qquad \tilde{H}_{\mu\nu} = \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma}H^{\rho\sigma};$$

where $E_{\mu\nu}$ and $H_{\mu\nu}$ are electric and magnetic field tensors respectively, obtained independently from the independent four potentials A_μ and B_μ. A_μ is the four potential associated with electric charge while B_μ is related to magnetic charge [23,24]. $\varepsilon_{\mu\nu\rho\sigma}$ is antisymmetric 4-index Levi-Civita symbol. (~) symbol represents dual part and other symbols have their usual meanings. The components of $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ are expressed as :

$$F_{oj} = E^j, \qquad F_{jk} = \varepsilon_{jkl}H^l;$$
$$\tilde{F}_{oj} = -H^j, \qquad \tilde{F}_{jk} = \varepsilon_{jkl}E^l; \tag{3}$$

where E and H are the generalized electric and magnetic fields of dyons [24]. These fields are given by

$$E = -\partial A/\partial t - \nabla\phi_e - \nabla\times B,$$
$$H = -\partial B/\partial t - \nabla\phi_g + \nabla\times A, \tag{4}$$

where $\{A^\mu\} = \{\phi_e, A\}$ and $\{B^\mu\} = \{\phi_g, B\}$. The relativistic notations $\{x^\mu\} = \{t, r\} = \{t,x,y,z\}$ and signature (+,–,–,–) are used throughout the text. Natural units are also used throughout the notations. Generalized electromagnetic fields (4) satisfy the symmetrized Maxwell-Dirac equation [24] where tensorial form is expressed in the following covariant notation.

$$F_{\mu\nu,\nu} = j_\mu, \qquad \tilde{F}_{\mu\nu,\nu} = k_\mu, \tag{5}$$

where $\{j^\mu\} = \{\rho_e, j\}$ and $\{k^\mu\} = \{\rho_m, k\}$ are respectively electric and magnetic four current densities. Generalized Maxwell's equations (5) are invariant under duality transformations.

$$(F_{\mu\nu}, \tilde{F}_{\mu\nu}) = (F_{\mu\nu}\cos\theta + \tilde{F}_{\mu\nu}\sin\theta, F_{\mu\nu}\sin\theta - \tilde{F}_{\mu\nu}\cos\theta),$$
$$(j_\mu, k_\mu) = (j_\mu\cos\theta + k_\mu\sin\theta, j_\mu\sin\theta - k_\mu\cos\theta), \tag{6}$$

where $$g/e = B_\mu/A_\mu = k_\mu/j_\mu = -\tan\theta. \tag{7}$$

As such, the two potentials A_μ and B_μ are considered independently and related together only by eq. (7). Since eqs. (4) are defined in terms of potential A_μ and B_μ, so the electric and magnetic fields (of dyons) are not taken independently like the ordinary electromagnetic fields.

The principle of stationary action states that the entire action I_ρ, given by eq. (1) must be invariant under infinitesimal variation in dynamical variables *i.e.*

$$
\begin{aligned}
x^\mu(\tau) &\to x^\mu(\tau) + \delta x^\mu(\tau), \\
A_\mu(x) &\to A_\mu(x) + \delta A_\mu(x), \\
B_\mu(x) &\to B_\mu(x) + \delta B_\mu(x), \\
\delta x_\mu(\tau) &\to 0 \text{ for } |\tau| \to \infty. \\
\delta A_\mu(x) &\to 0 \text{ for } |x| \to \infty. \\
\delta B_\mu(x) &\to 0 \text{ for } |x| \to \infty.
\end{aligned} \tag{8}
$$

After taking these variations into account in eqs. (1), we get the following dynamical equations for generalized electromagnetic fields of a dyon in prescence of gravitation *i.e.*

$$
\sum_n m_n \left[d^2 x_n^\mu / d\tau^2 + \Gamma^\mu_{\nu\lambda} \left(dx_n^\nu / d\tau\right)\left(dx_n^\lambda / d\tau\right)\right]
$$

$$
= \sum_n \left[e_n F_{\mu\nu} (dx_n^\nu / d\tau) + g_n \tilde{F}_{\mu\nu} (dx_n^\nu / d\tau)\right], \tag{9}
$$

$$
\left[d\tau = \left(-g_{\mu\nu}(x) dx_n^\mu dx_n^\nu\right)^{1/2}\right]
$$

$$
\partial / \partial x^\mu \left\{(-g)^{1/2} F^{\mu\nu}(x)\right\} = -\sum_n e_n \int \delta^4 (x - x_n).(dx_n^\nu / d\tau) d\tau, \tag{10}
$$

$$
\partial / \partial x^\mu \left\{(-g)^{1/2} \tilde{F}^{\mu\nu}(x)\right\}^{\cdot} = -\sum_n g_n \int \delta^4 (x - x_n).(dx_n^\nu / d\tau) d\tau, \tag{11}
$$

where the four-velocity is defined as :

$$
v_n^\mu = dx_n^\mu / d\tau \tag{12}
$$

and $\Gamma^\mu_{\nu\lambda}$ are the components of Christofell's symbol.

Taking the summation convention into account, eq. (9) reduces to

$$
m\left[d^2 X_n^\mu / d\tau^2 + \Gamma^\mu_{\rho\nu} \upsilon_n^\nu \upsilon_n^\rho = \left[e_n F_{\mu\nu} + g_n \tilde{F}_\mu^\nu\right] v_n^\nu, \tag{13}
$$

which is the Lorentz force equation of motion for a charged particle (dyon) moving in a generalized electromagnetic and gravitational fields. In the absence of gravitation the Lorentz force equation of motion for dyons is obtained (from eq. (13)) as

$$
m.d^2 X_n^\mu / d\tau^2 = \left[e_n F_{\mu\nu} + g_n \tilde{F}_{\mu\nu}\right] v_n^\mu, \tag{14}
$$

which reduces to $m.dv/dt = e[\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{H}] + g[\boldsymbol{H} - \boldsymbol{V} \times \boldsymbol{E}]$. (15)

As the two potentials A_μ and B_μ are varied independently, the eq. (15) clearly states that it has nothing to do with potentials but is related only to electric and magnetic fields. The definitions of electric and magnetic fields are of course taken from eq. (4). Thus one should not confused these electric and magnetic fields with ordinary electromagnetic fields. Relations of A_μ and B_μ with electric and magnetic fields clearly state that these two potentials give rise to generalized equations of motion (15) for dyons. It is clear that the two potentials are taken independently and the nature of eq. (15) comes only due to the definitions of new electric and magnetic fields (given by eq. (4)) described in terms of electric and magnetic four potentials. In the absence of magnetic charge, we obtain the following equation of motion for a charged particle moving in electromagnetic and gravitational fields :

$$m\left[(dv^\mu / dt) + \Gamma^\mu_{\nu\rho} v^\nu U^\rho\right] = eF_{\mu\nu} v^\nu. \tag{16}$$

Equation (13) reduces to usual geodesic equation for arbitary gravitational field. As such eq. (13) is considered as a generalized equation of motion for a charged particle (dyon) interacting with electromagnetic field (free from string singularities) in presence of gravitation. Defining the electric and magnetic four-current for a charged particle (dyon) as

$$j^\mu(x) = -\sum_n e_n \int \delta^4(x - x_n).v^\mu_n d\tau, \tag{17}$$

$$\kappa^\mu(x) = -\sum_n g_n \int \delta^4(x - x_n)\, v^\mu_n d\tau, \tag{18}$$

and inserting these expressions into eqs. (10) and (11) we get

$$\partial / \partial x^\mu \left[(-g)^{1/2} F_{\mu\nu}(x)\right] = j^\nu(x),$$

$$\partial / \partial x^\mu \left[(-g)^{1/2} \tilde{F}_{\mu\nu}(x)\right] = k^\nu(x), \tag{19}$$

which are reduced to following covariant forms of eq. (5)

$$F_{\mu\nu,\nu} = \nabla^\nu F_{\mu\nu} = j_\mu \quad \tilde{F}_{\mu\nu,\nu} = \nabla^\nu \tilde{F}_{\mu\nu} = k_\mu, \tag{20}$$

where $\{\nabla^\nu\} = \{\nabla^0, \nabla^1, \nabla^2, \nabla^3\}$ is the covariant derivative. The energy momentum tensor for a material system described by an invariant action I_p can now be expressed as :

$$T^{\mu\nu}(x) = (-g)^{1/2} \sum m_n \int^{+\infty} d\tau (dx^\mu_n / d\tau).(dx^\nu_n / d\tau)\delta^4(x - x_n$$

$$+ F^\mu_\rho(x) F^{\rho\nu}(x) + \tilde{F}^\mu_\rho(x)\tilde{F}^{\rho\nu} - \frac{1}{4} g^{\mu\nu}(x) F_{\rho\sigma}(x) F^{\rho\sigma}(x)$$

$$- \frac{1}{4} g^{\mu\nu}(x) \tilde{F}_{\rho\sigma}(x) \tilde{F}^{\rho\sigma}(x), \tag{21}$$

which leads to the following equation after taking the proper variation with respect to field variables :

$$\partial / \partial x^{\nu}\left[\sqrt{g}.T_{\lambda}^{\nu}-\frac{1}{2}(\partial g_{\mu\nu} / \partial x^{\lambda})\sqrt{g}T^{\mu\nu}\right] = (T_{\lambda}^{\nu}); \nu=0. \tag{22}$$

Thus, the energy momentum tensor eq. (21) for dyons in presence of gravitation is conserved (in the covariant form) if and only if the action of the matter is considered as an invariant scalar. Also with Ip as scalar energy momentum tensor (21) is symmetric in its covariant indices μ and ν. This proves that general covariance implies energy momentum conservation like the guage invariance implies charge conservation. If we include the pure gravitational action I_G (in terms of curvature scalar) with Ip, the total action is defined as :

$$I = Ip + I_G, \tag{23}$$

where $$I_G = I/(16\pi G)\int(-g)^{1/2} R(x)d^4x \tag{23a}$$

and $R(x)$ is curvature scalar and G is gravitational constant. The total action (23) immediately leads to following Einstien's field equation,

$$R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R+8\pi GT^{\mu\nu} = 0 \tag{24}$$

and bianchi identity $$\nabla_{\mu}(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R) = 0 \tag{25}$$

3. Linear gravity : as the symmetry of electromagnetism in presence of sources

For convenience, the coordinate system to be used in linear theory of gravity is cartesian and hence the Lorentz metric takes the form $\eta_{\mu\nu} = \eta^{\mu\nu} = (1, -1, -1, -1)$ in natural units. The gravitational field described by the metric $g_{\mu\nu}$ is considered as weak field and defined as

$$g_{\mu\nu} = \eta_{\mu\nu} + t_{\mu\nu}, \tag{26}$$

with the approximation when the field is sufficiently weak, $|t_{\mu\nu}| \ll 1$. In a coordinate system in which the metric tensor is Lorentz metric, the four velocity is defined as

$$v^{\mu} = dx^{\mu}/dt = \gamma(1, v) \text{ where } \gamma = (1-v^2)^{-1/2}.$$

If the particle is moving very slowly so that $v \ll 1$ then up to the good approximation we may write

$$\Gamma_{\alpha\beta}^{\mu}(dx^{\alpha}/d\tau)(dx^{\beta}/d\tau) \approx \Gamma_{00}^{\mu}+2\Gamma_{0j}^{\mu}v^{j}, \tag{27}$$

where $$\Gamma_{\alpha\beta}^{\mu} = g^{\mu\sigma}\Gamma_{\sigma\alpha\beta} \tag{28}$$

and $$\Gamma_{\sigma 0\beta} = \frac{1}{2}(\partial_{\beta}h_{0\sigma}+\partial_{0}h_{\sigma\beta}-\partial_{\sigma}h_{0\beta}) \tag{29}$$

we see that for static fields, the terms $\partial_0 h_{\beta\sigma} = 0$ and $\Gamma_{\sigma 0\beta} = -\beta_{\beta 0\sigma}$ suggest that $\Gamma_{\sigma 0\beta}$ can be divided into antisymmetric and symmetric part and we write

$$\Gamma_{\sigma 0\beta} = \Im_{\sigma\beta} + \frac{1}{2}\partial_0 h_{\sigma\beta} \tag{30}$$

where
$$\Im_{\sigma\beta} = \frac{1}{2}(\partial_\sigma h_{0\beta} - \partial_\beta h_{0\sigma}) = -\Im_{\beta\sigma} \tag{31}$$

Dropping the second term in the right hand side of equation (30) and raising the index with $\eta^{\mu\sigma}$ we find

$$\Gamma^{\mu}_{0\beta} = \frac{1}{2}\begin{bmatrix} 0 & -2G_x & -2G_y & -2G_z \\ -2G & 0 & -M_z & -M_y \\ -2G_y & M_z & 0 & -M_x \\ -2G_z & M_y & M_x & 0 \end{bmatrix}. \tag{32}$$

Eq. (31) immediately satisfies the following differential equation

$$\partial_\mu \Im_{\sigma\beta} + \partial_\sigma \Im_{\beta\mu} + \partial_\beta \Im_{\beta\sigma} = 0 \tag{33}$$

and gives rise (for μ, β, σ = 1, 2, 3) to $\nabla . M = 0$ and $\nabla \times G = -\frac{1}{2}\partial M / \partial t$. (34)

Except for the factor half on right hand side of (34), these equations are the same as the homogeneous Maxwell's equations with G and M playing the role of ordinary electric and magnetic fields of electrodynamics. Substituting eq. (27) into eq. (16) and using eq. (32) we get

$$md^2 x/dt^2 = e[\boldsymbol{E} + \boldsymbol{v}\times \boldsymbol{H}] + m[\boldsymbol{G} + \boldsymbol{v}\times \boldsymbol{H}]. \tag{35}$$

Thus gravitational force closely resembles to electromagnetic force. If we incorporate monopole into account eq. (13) takes the form

$$md^2 x/dt^2 = e[\boldsymbol{E} + \boldsymbol{v}\times \boldsymbol{H}] + g[\boldsymbol{H} - \boldsymbol{v}\times \boldsymbol{E}] + m[\boldsymbol{G} + \boldsymbol{v}\times \boldsymbol{M}]. \tag{36}$$

which is the nonrelativistic equation of motion for particles carrying simultaneously electric, magnetic and gravitational charges (masses). The close resemblance between the equations for $\boldsymbol{G}$ and $\boldsymbol{M}$ and Maxwell's equation for static fields suggests the names "gravito-electric (or gravielectric) field" for $\boldsymbol{G}$ and "gravito-magnetic (or gravimagnetic) field" for $\boldsymbol{M}$. Other two equations for $\nabla . \boldsymbol{G}$ and $\nabla \times \boldsymbol{M}$ may directly be obtained from linearizing Einstein-Maxwell's equation. As such the linear gravity theory (Maxwellian gravity) resembles with classical electromagnetic theory. Thus, eq. (31) is the weak field approximation (in non-relativistic limit) of eq. (13) described as the unified equation of motion for gravitation and generalized electromagnetism. As such, eq. (23), the combined action, follows the equation of motion (36) :

$$\nabla . \boldsymbol{G} = -\rho; \qquad \nabla \times \boldsymbol{G} = -\partial \boldsymbol{M} / \partial t;$$
$$\nabla . \boldsymbol{M} = 0; \qquad \nabla \times \boldsymbol{M} = -\boldsymbol{J}_G + \partial \boldsymbol{G} / \partial t. \tag{37}$$

These equations lead to asymmetry in gravielectric and gravimagnetic fields and are also not invariant under duality transformation like eq. (6).

Postulation of gravimagnetic monopole (Heavisidian monopole) [17–21] gives rise to symmetrized Maxwell's equations for linear gravitational fields :

$$\nabla.\boldsymbol{G} = -\rho_G; \qquad \nabla\times\boldsymbol{G} = -\partial\boldsymbol{M}/\partial t+\boldsymbol{J}_H;$$
$$\nabla.\boldsymbol{M} = -\rho_H; \qquad \nabla\times\boldsymbol{M} = -\boldsymbol{J}_G+\partial\boldsymbol{G}/\partial t. \tag{38}$$

where ρ_G is the gravitational charge (mass) density and ρ_H is gravi-magnetic (Heavisidian) charge (mass). To surmount the difficulties faced by Dirac-Vito here let us introduced [23–25] the idea of two four-potentials $\{a_\mu\}$ and $\{b_\mu\}$ associated with gravitational (gravi-electric) and Heavisidian (gravi-magnetic) charges (masses) respectively. Thus, we obtain the following co-variant tensorial forms of eq. (38) :

$$f_{\mu\nu,\nu} = J_\mu^{(G)}; \; \tilde{f}_{\mu\nu,\nu} = J_\mu^{(M)}; \tag{39}$$

where
$$f_{\mu\nu} = \varepsilon_{\mu\nu}-t_{\mu\nu}; \; \tilde{f}_{\mu\nu} = t_{\mu\nu}+\tilde{\varepsilon}_{\mu\nu}; \tag{40}$$
$$\varepsilon_{\mu\nu} = a_{\mu\,\nu}-a_{\nu\,\mu}; \; t_{\mu\nu} = b_{\mu\,\nu}-b_{\nu,\mu};$$
$$\{a_\mu\} = \{\phi^{(G)},\boldsymbol{a}\} \qquad \text{(gravielectric four potential)},$$
$$\{b_\mu\} = \{\phi^{(M)},-\boldsymbol{b}\} \qquad \text{(gravimagnetic four potential)},$$
$$\{J_\mu^G\} = \{\rho^{(G)},-\boldsymbol{J}_G\} \qquad \text{(gravielectric four current)},$$
$$\{J_\mu^M\} = \{\rho^{(M)},-\boldsymbol{J}_M\} \qquad \text{(gravimagnetic four current)}.$$

The gravielectric and gravimagnetic field of linear gravity described in eq. (38) are now expressed as the generalised field of gravito-dyons (components of $f_{\mu\nu}$ and $\tilde{f}_{\mu\nu}$) in the following manner :

$$\boldsymbol{G} = \partial\boldsymbol{a}/\partial t+\nabla\phi_g+\nabla\times\boldsymbol{b}$$
$$\boldsymbol{M} = -\partial\boldsymbol{b}/\partial t+\nabla\phi_M-\nabla\times\boldsymbol{a} \tag{41}$$

Eq. (39) are then expressed as the dual invariant and symmetrized Maxwell's Dirac Equation of gravitodyons (in presence of gravielectric and gravimagnetic charges). Thus, the equation of motion for a particle carrying simultaneous existence of gravielectric and gravimagnetic charges (masses) be described as

$$(m+h)d\boldsymbol{v}/dt = m(\boldsymbol{G}+\boldsymbol{v}\times\boldsymbol{H})+h(\boldsymbol{M}-\boldsymbol{v}\times\boldsymbol{G}). \tag{42}$$

On the other hand, the generalised equations of motion of a particles carrying simultaneously electric, magnetic, gravitational and Heavisidian (gravimagnetic) charges (masses) may be obtained on linearizing eq. (13) and introducing gravimagnetic poles therein as ·

$$(m+h)d\boldsymbol{v}/dt = e[\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{H}]+g[\boldsymbol{H}-\boldsymbol{v}\times\boldsymbol{E}]+m[\boldsymbol{G}+\boldsymbol{v}\times\boldsymbol{M}]+h[\boldsymbol{M}-\boldsymbol{v}\times\boldsymbol{G}]. \tag{43}$$

4. Charge (mass) quantization and angular momentum

Various authors [9,10,15,16] obtained the charge quantization condition by introducing concept of particles that possess both electric and magnetic charges (or dyons). Following the same lines let us introduce the gravitational dyons (particles that possess both gravi-electric and gravi-magnetic charges). Using the equation of motion (42) of gravitational charge (mass) in a generalized gravi-electromagnetic fields, we may write the following expression for the angular momentum vector of j-th gravito-dyon of mass (m_j, h_j) moving in the field of k-th gravito-dyon of mass (m_k, h_k) which is assumed at rest

$$\boldsymbol{J} = \boldsymbol{r} \times \boldsymbol{p} + (m_j h_k - m_k h_j)\boldsymbol{r}/r, \tag{44}$$

where $\boldsymbol{p} = (m + h)d\boldsymbol{r}/dt$.

This angular momentum is same as derived earliar [17] but can not be acceptable in the presence of monopole (gravimagnetic) because it is not gauge invariant. Thus we agree with the results of Rajput [26] that to make the angular momentum rotationally symmetric and gauge invariant, the complex behaviour of gravito-dyon structure is needed. Thus dyons are represented as a dual charged particles in two dimensional vector space. Real and imaginary part of gravitodyons are described as gravieletric and gravimagnetic charges (masses)

Thus the generalized charge (mass), four potential, four-current, and fields of gravito dyons may be described respectively as :

$$q = m - ih; \qquad v_\mu = a_\mu - ib_\mu; \tag{45}$$

$$J_\mu = J_\mu^G - Ik_\mu^m; \qquad \Psi = \boldsymbol{G} - i\boldsymbol{M}. \tag{46}$$

As such, the interaction of j-th dyon in the field of k-th dyon is described (from the interaction part $v_\mu J^\mu$ of Lagrangian density [23] as :

$$I_{jk} = \left(a_\mu^{(k)}/e_k\right) q_k q^j v_\mu^{(j)}. \tag{47}$$

where $$q_k q_j = q_j q_k = (m_j m_k + h_j h_k) + i(m_j h_k - h_j m_k). \tag{48}$$

Equation (47) shows that

(a) interaction between two gravito-dyons is zero when their generalized charges are ortho-gonal in their combined charge (mass) space.

(b) interaction depends on gravielectric coupling parameter $\alpha_{jk} = m_j m_k + h_j h_k$ under the constancy condition $m_j/h_j = m_k/h_k =$ constant.

(c) interaction depends on the gravimagnetic coupling parameter (*i.e.* Chirality) :

$\mu_{jk} = m_j h_k - h_j m_k$ under the constancy condition $m_j/h_j = - h_k/m_k$.

As such, the expression for gauge invariant and rotationally symmetric angular momentum of gravito-dyons now be written as :

$$J = \boldsymbol{r} \times \left[\boldsymbol{p} - \mu_{jk}\boldsymbol{v}^T\right] + \mu_{jk}\boldsymbol{r}/r. \tag{49}$$

where v^{T} is the transverse spatial part of generalized four-potential $\{v_{\mu}\}$ associated gravito dyons and we have used $\Psi = q_k r / r^3$ in obtaining the expressions (48) and (49). The commutation relations for the components of angular momentum (49) and gauge invariant linear momentum

$$\pi = p - \mu_{jk} v \tag{50}$$

are already derived by Rajput [26]. Equation (49) immediately gives rise a scalar quantity $r J/h = \mu_{jk}$ This quantity commutes with all observables Eq (49) also shows that there is a residual angular momentum

$$J_{\text{res}} = \mu_{jk} (r / r) \tag{51}$$

carried by generalized fields of generalized charges besides the orbital and spin angular momentum of each particle. If the angular momentum (49) is quantized along the line joining the generalized masses q_j and q_k of gravito-dyons following quantization condition

$$\mu_{jk} = n, \tag{52}$$

where n is the positive integer. Eq. (52) reduces to Chirality quantization condition

$$\mu_{jk} = m_j h_k - m_k h_j = 0, + 1, + 2,. \tag{53}$$

Eq. (52) reduces to following charge (mass) quantization (like Dirac quantization condition) for gravimonopole

$$mh = n, \tag{54}$$

where $q_j = (m,0)$ and $q_k = (0,h)$. Eq. (54) leads that the forces between gravimagnetic charges are enormous being the most interacting form of matter.

5. Discussion and conclusion

Starting from the invariant action principle (1) we have obtained the manifestly covariant equation of motion (13) in presence of gravitation and generalized electromagnetic fields of dyons. Generalized electromagnetism is contained in energy momentum tensor defined by eq. (21). Thus, Einstein's field equation (24) describes the generalized theory of gravitation and in presence of electric and magnetic charges, two four potentials A_{μ} and B_{μ} used in Section 2, are linearly independent of each other and give rise to generalized electric and magnetic fields E and H associated with dyons. The equation of motion (15) is described as the Lorentz force equation in which electric and magnetic fields are not the conventional fields but are the generalized electric and magnetic fields of dyons. Thus for dyons, the equation of motions relates two independent potentials with the dependent electric and magnetic fields.

Heavisidian fields were postulated in 1893 as the analogue of magnetic fields in linear gravity. Many authors worked in this direction and suggested that this type of fields is obtained as the motion of moving mass (charge) and plays the role of magnetic fields. So we refer the Heavisidian fields as gravimagnetic fields of linear gravity. The gravimagnetic field M is obtained from the Christofell's symbol in weak field approximation. Postulation of gravimagnetic monopole gives rise to symmetrized Maxwell's eq. (38) and describe the structural symmetry between generalized eletromagnetic and generalized gravielectromagnetic fields of dyons. Equation (42) is the equation of motion for a particle carrying simultaneous existence of gravielectric and gravimagnetic charges (masses). On the other hand, eq. (43) is described as the unified equation of motion for the generalized fields of four charges named as electric, magnetic, gravitational and gravi-magnetic charges. The rotationally symmetric gauge invariant angular momentum operator associated with generalized fields of gravito-dyons is defined by eq. (49). For quantization of angular momentum, it is necessary to obtain the quantization condition (52) which further reduces to the Chirality quantization condition (53). Although the gravi-magnetic monopoles are not found experimentally, the consideration of hypothetical mass h leads to the quantization condition (54). If we assume that the fundamental gravitational charge is equal to the rest mass of an electron, the gravitaional charge quantization condition (54) then leads $Gm_e h = n$ or $Gh^2 = n^2/Gm_e^2$, where G is gravitational constant. Now, on proper substitution, we find $Gh \approx 5.7 \times 10^{44}$. Thus, it is clear that the forces between gravimagnetic charges are enormous and these charges are the most strongly interacting form of matter. This enormous amount of mass may lead to the conclusion that this is probably one of the reasons that Heavisidian (gravimagnetic) poles are not found yet.

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